

Metastable States in CA models for Traffic Flow

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Abstract. Measurements of traffic flow show the existence of metastable states of very high throughput. These observations cannot be reproduced by the CA model of Nagel and Schreckenberg (NaSch model), not even qualitatively. Here we present two variants on the NaSch model with modified acceleration rules ('slow-to-start' rules). Although these models are still discrete in time and space, different types of metastable states can be observed.

1 Introduction

Simulation results of the CA model for traffic flow introduced by Nagel and Schreckenberg [1] (NaSch model, for a detailed explanation see [1,2]) show a reasonable agreement with experimental data, although the model definition includes only a few simple update rules. Due to its simplicity it can be used very efficiently for computer simulations, but also an analytical treatment is possible [2].

One important experimental result [3] which cannot be gained from the NaSch model is the occurrence of metastable states near the density of maximum flow. Very recently such states have been found in a modified version of the NaSch model where a limited braking capacity of the cars and a continuous space is considered [4]. The continuous space allows very small amplitudes of the fluctuations compared to the discrete NaSch model. Therefore one could ask if a continuous space is necessary to obtain metastable states.

Here we present the results of a numerical investigation of two variants on the NaSch model with modified acceleration rules, which are still discrete in time and space. First, we assigned a larger braking probability to stopped cars and second, we analysed the modification suggested by Takayasu and Takayasu [5]. It turns out that metastable states can be found in both cases.

2 NaSch model with 'slow-to-start' rule

In this part of the investigation we consider the NaSch model with maximum velocity $v_{max} = 5$ and braking probability $p = 0.01$ of the moving cars. In

contrast to the original model, a higher value $p_s = 0.5$ of the braking probability is assigned to stopped cars.

We performed simulation runs for periodic systems containing $L = 10000$ lattice sites. This system size is sufficient to exclude finite size effects for the chosen set of parameters. Fig. 1 shows the fundamental diagram of the modified

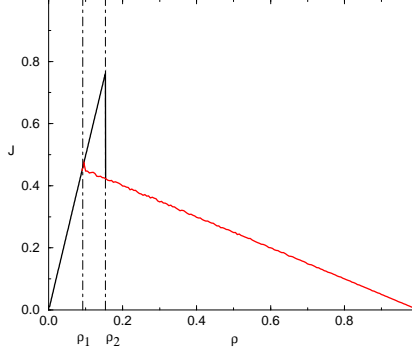


Fig. 1. Fundamental diagram of the modified NaSch model ($v_{max} = 5$, $p_s = 0.5$, $p = 0.01$, $L = 10000$).

model. Obviously the average flow $J(\rho)$ can take two values in the density interval between ρ_1 and ρ_2 dependent on the chosen initialisation. The larger values of the average flow can be obtained using a homogeneous initialisation of the system. The lower branch is obtained starting from a complete jammed state. Moreover varying the particle number adiabatically, one can trace a hysteresis loop. One gets the upper branch by adding cars to the stationary state with $\rho < \rho_1$ and the lower one by removing cars from the stationary state with $\rho > \rho_2$. For a fixed value of p , $\Delta J = J(\rho_2) - J(\rho_1)$ depends linearly on p_s for wide a range of parameters.

The microscopic structure of the jammed states in the modified model differs from those found in the NaSch model. While jammed states in the NaSch model contain with an exponential size-distribution [2], one can find phase separation in the modified model. The reason for this behaviour is the reduction of the outflow from a jam. Therefore the density in the free flow regime is smaller than the density of maximum flow and cars can propagate freely in the low density part of the lattice. Due to the reduction of the density in the free flow regime, no spontaneous formation of jams is observable in the stationary state, if fluctuations in the free flow regime are rare.

This picture can be confirmed by a simple phenomenological approach. Obviously the flow in the homogeneous branch is given by $J_h = \rho(v_{max} - p) = \rho v_f$, because every car can move with its desired velocity v_f . Assuming that the high density states are phase separated, we can obtain the second branch of the fun-

damental diagram. The phase separated states consist of a large jam and a free flow regime, where each car moves with velocity v_f . The density in the free flow regime ρ_f is determined by the average waiting time $T_w = \frac{1}{1-p_s}$ of the first car in the jam and v_f , because neglecting interactions between cars the average distance of two consecutive cars is given by $\Delta x = T_w v_f + 1 = \rho_f^{-1}$. Using the normalisation $L = N_J + N_F \Delta x$ ($N_{F(J)}$ number of cars in the free flow regime (jam)) we find that the flow is given by

$$J_s(\rho) = (1 - \rho)(1 - p_s). \quad (1)$$

Obviously ρ_f is precisely the branching density ρ_1 , because for densities below ρ_f the jamlength is zero. It should be noted that this approach is only valid if ($p_s \gg p$) and $v_{max} > 1$ holds. Small values of p have to be considered in order to avoid interaction between cars due to the velocity fluctuations and $v_{max} > 1$, because otherwise cars can stop spontaneous in the free flow regime and therefore initiate a jam.

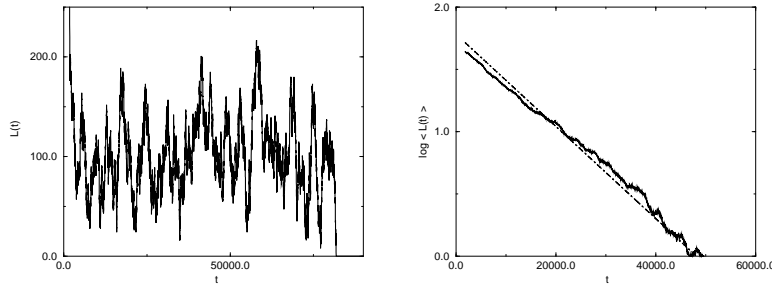


Fig. 2. Time-dependence of the jamlength for $\rho = 0.095$. The left part of the figure shows the time evolution of the length $L(t)$ of one sample. The average over 10000 samples (right part of the figure) shows that the jamlength decays exponentially.

Measurements of the average flow show that the lower branch of the fundamental diagram is not stable near the density ρ_1 . Therefore we performed a more detailed stability analysis of the homogeneous and the jammed state near ρ_1 and ρ_2 . Near ρ_1 the large jam present in the initial configuration resolves, the average length decays exponentially in time (Fig. 2). It should be noted that this behaviour is not the consequence of a continuous "melting" of the large jam. In contrast, the jamlength is strongly fluctuating without any systematic time-dependence. Once a homogeneous state without a jammed car is reached, no jam appears again. Therefore the homogeneous state is stable near ρ_1 .

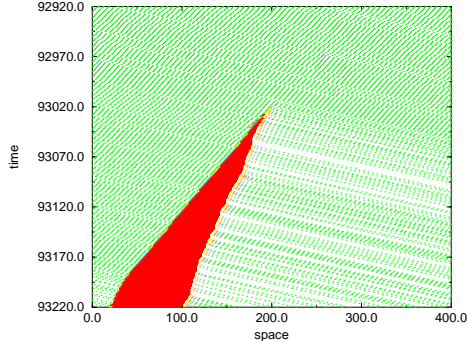


Fig. 3. Space-time diagram of the modified NaSch model for $\rho = 0.15$, $L = 400$, $p = 0.01$ and $p_s = 0.5$. The homogeneous initial state is not destroyed immediately, but after approximately 90000 lattice updates. In the outflow regime of the jam the density is reduced compared to the average density.

Analogous to the metastable jammed states near ρ_1 , homogeneous initialisations for densities slightly above ρ_2 lead to metastable homogeneous states with short lifetimes. Fig. 3 shows the spontaneous formation of jams due to velocity fluctuations. The finite lifetimes of the homogeneous states are the qualitative difference between this model and the cruise-control limit [6], where the time evolution of homogeneous states at low densities is completely deterministic.

3 T^2 model

Takayasu and Takayasu (T^2) [5] suggested a cellular automaton model with another type of a slow-to-start rule, which is defined as follows: A standing car with exactly one empty cell in front of it accelerates with probability $q_t = 1 - p_t$, while all other cars accelerate deterministically. The other update rules of the NaSch model are unchanged. Due to this modification already for $v_{max} = 1$ the particle-hole symmetry is broken. In Fig. 4 we show the fundamental diagram for $v_{max} = 1$, $p = 0.5$ and $p_t = 1$, i.e. stopped cars can only move if there are at least two empty cells in front of it. Obviously completely blocked states exist for densities $\rho \geq 0.5$, where the number of empty cells in front of each car is smaller than two. Since fluctuations are absent in those states, they have an infinite lifetime. Therefore the flow in the stationary state is zero.

Although the states with a finite flow are not stationary, typically one has to perform an extremely large number of update steps until the flow vanishes for large system sizes and densities slightly above $\rho = 0.5$, because the number of blocked configurations is very small compared to the total number of config-

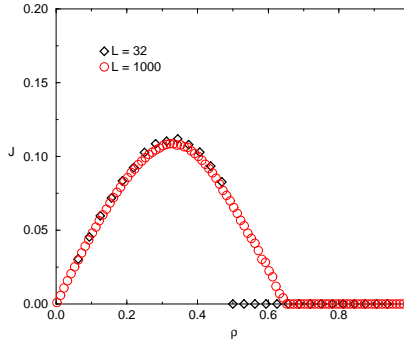


Fig. 4. Fundamental diagram of the T^2 model ($v_{max} = 1$, $p = 0.5$, $p_t = 1$) for two system sizes. For densities slightly above $\rho = 0.5$ the stationary state could only be reached for the small system.

urations. Precisely at $\rho = 0.5$, the blocked state is unique and the typical time to reach this state diverges exponentially with the system size. Therefore we used very small systems in order to obtain the lower branch of the fundamental diagram.

Another interesting feature is the form of the fundamental diagram. There is some experimental evidence that in certain situations the shape of the fundamental diagram differs from the standard convex form. This behaviour of the average flow can be easily obtained tuning the parameter p_t [2,7].

4 Summary

The numerical analysis of two modifications of the NaSch model shows that metastable states can also be found in CA models for traffic flow.

The NaSch model with 'slow-to-start' rule shows the coexistence of phase separated and homogeneous states in a density interval near the density ρ_2 of maximum flow. Near ρ_2 interactions between cars become important and one can find spontaneous formation of jams. Therefore the reduction of the density in the outflow regime of a jam leads to stable phase separated states. The reduction of interactions between cars in the free flow regime can be confirmed by a phenomenological approach, which gives very accurate results. In contrast to the cruise-control limit of the NaSch model [6], where also metastable states can be found, fluctuations are present in both coexisting states. Finally we mention that this modified version of the NaSch model can reproduce the fundamental diagram of the spacecontinuous approach by Krauß et. al. [4].

In the $p_t = 1$ limit of the T^2 model we find metastable states for densities beyond $\rho = 0.5$. For $\rho \gtrsim 0.5$ states with a finite flow have very large lifetimes, but if the system reaches a blocked state these states are stable because of the absence

of fluctuations. Therefore the $p_t = 1$ limit of the T^2 model is complementary to the cruise-control limit of the NaSch model, where one finds the absence of fluctuations for homogeneous states at low densities.

These modifications of the NaSch model show that CA models for traffic can reproduce a broad spectrum of experimental results although the update rules oversimplify the individual behaviour of the drivers.

References

1. Nagel K., Schreckenberg M.: *A cellular automaton model for freeway traffic*. J. Physique **I2**, 2221 (1992)
2. Schadschneider A.: *Analytical approaches to cellular automata for traffic flow: Approximations and exact solutions*. These proceedings
3. Kerner B.S., Rehborn H.: *Experimental properties of complexity in traffic flow*. Phys. Rev. **E53**, R4275 (1996)
4. Krauß S., Wagner P., Gawron C.: *Metastable states in a microscopic model of traffic flow*. Phys. Rev. **E55**, 5597 (1997)
5. Takayasu M. and Takayasu H.: *1/f noise in a traffic model*. Fractals **1**, 860 (1993)
6. Nagel K., Paczuski M.: *Emergent traffic jams*. Phys. Rev. **E51**, 2909 (1995)
7. Schadschneider A., Schreckenberg M.: *Traffic flow models with 'slow-to-start' rules*. Ann. Physik **6**, 541 (1997)